TOWARDS AN IMPROVED STONE MINE PILLAR DESIGN METHODOLOGY: OBSERVATIONS FROM A MISTAKE

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ABSTRACT

The mining engineering design professional has limited practical and reliable tools for planning successful room-and-pillar stone mines using readily-available and collectible information. Three techniques are in common use today: the hard rock CANMET method of Hedley and Grant, the hard rock method of Stacey and Page, and the oil shale method of Hardy and Agapito. Other methods have been proposed, such as the USBM method of Obert and Duvall, the CSIR/Penn State method of Bieniawski, and the soft rock confined core method of Abel, Wilson, and Ashwin. However, the latter have practical shortcomings when applied to room-and-pillar stone mines such as developed for construction aggregate production. Ideally, the use of multiple techniques resulting in the same acceptable and reliable answer is the goal. Recently, in several underground stone mines areas of undersized pillars were examined. These undersized pillars apparently resulted from non-adherence to a mine plan. These undersized pillars now exhibit strong evidence of incipient failure such as slabbing, opening of through-going fractures, and hour-glassing. This situation allows examination of pillar designs at a "safety factor" of essentially one. This rare opportunity allowed the examination of the suitability and adjustment of the first three design methods discussed, resulting in greater overall confidence in the methodologies.

INTRODUCTION

There are no generally accepted deterministic (calculational) methods for underground mine pillar design in hard rock. In this paper, we differentiate between "hard rock" and coal and evaporites, sometimes termed "soft rock." "Hard rock" can be considered to include most igneous and metamorphic rocks and well-indurated sedimentary rocks such as limestones, dolomites, and sandstones. The subject of this paper is hard-rock pillar design. Pillars are composed of the intact rock substance, such as is tested in the laboratory from cores or pieces extracted in the field and the naturally-occurring discontinuities, such as fractures, joints, and bedding planes. The presence of discontinuities results in an overall pillar strength (rock mass strength) considerably less than the laboratory strength. Rock mass strength is further reduced by the process of excavation by blasting, which damages and loosens the rock at the blasted surface. Even when excavated by purely mechanical means, such as mechanical miners or rippers, the rock mass relaxes, dilates, and loosens from the load reduction.

Over the years, a number of methodologies have been developed to calculate full-scale room-and-pillar mine pillar strengths based upon laboratory-scale strengths obtained from diamond-drill core or saw-cut specimens. For square pillars, a relationship developed usually follows the form:

$$\sigma_p = C_0\,(w^a\,h^b) \qquad \text{(Eqn. 1)}$$

where
- σ_p = Pillar strength
- C_0 = Unconfined compressive strength of rock in the laboratory
- w = Pillar width
- h = Pillar height
- a,b = Constants found from field observations or laboratory experiments

Numerous publications describe, in detail, derivation of pillar strength formulae, especially for coal, and the interested reader is directed to rock mechanics texts and the papers referenced in the discussion below.

To overcome obstacles to design arising from the profound difference between laboratory specimens and in-situ mine pillars, several different methods of pillar design are in use in hard rock mining throughout the United States, Canada, and South Africa. However, all methods have several common factors:

- Use of laboratory-determined intact rock strength
- Use of empirically-derived (experience-based) factors to discount the rock strength to a rock mass strength
- Use of geometric-shape factors to adjust pillar strengths

I have used several different methods in concert, and recognizing which methods are more or less appropriate for the situation, seek to obtain an agreement among the method results, yielding a high level of confidence. The individual methods are described below.

In each case, the pillar stress due to overburden for flat-lying formations with a regular pattern of square or rectangular pillars is:

$$\sigma_v = \frac{(\gamma / 144) D}{1 - E} \qquad \text{(Eqn. 2)}$$

where σ_v = Vertical stress in pillar (psi)
γ = Unit weight of overburden (pcf)
D = Depth (ft) to pillar roof level
E = Extraction ratio where

$$E = 1 - \left[\frac{wl}{(w + R)(l + C)} \right]$$

where w = Pillar width (ft)
l = Pillar length (ft)
R = Room width (ft)
C = Crosscut width (ft)

HEDLEY AND GRANT METHOD

Hedley and Grant (1) proposed a pillar design method for very hard rock, such as found in the former uranium mines in Ontario, Canada. The formulation for square pillars is:

$$\sigma_p = \beta C_0 \left(w^a / h^b \right) \qquad \text{(Eqn. 3)}$$

where σ_p = Pillar strength
β = Discount factor to obtain estimated strength of a 12-inch-cube specimen from test data on smaller diamond-drill cores (here taken at 0.70 for a nominal 2-inch diameter core)
C_0 = Unconfined compressive strength in the laboratory of a cylindrical rock core
w = Pillar width
h = Pillar height
a = 0.5
b = 0.75

and the pillar stress due to overburden is calculated using Equation 2.

The pillar strength is then compared to the pillar stress as in

$$SF_{Hedley-Grant} = \sigma_p / \sigma_v \qquad \text{(Eqn. 4)}$$

where $SF_{Hedley\text{-}Grant}$ = Factor of safety in crushing for the Hedley-Grant Method
σ_v = Pillar stress calculated using Equation 2

STACEY-PAGE METHOD

Stacey and Page (2) in South Africa have proposed a pillar design method specifically developed for large-span excavations in hard rock. The formulation is:

$$\sigma_p = k \left(W_{eff}^{\;0.5} / h^{0.7} \right) \qquad \text{(Eqn. 5)}$$

where σ_p = Pillar strength in MPa (1 MPa = 145 psi)
k = Design rock mass strength in MPa, which is calculated by a series of steps using a process given in Stacey and Page (2) which results in a reduction factor of approximately 50% to 60% for many limestones and dolomites
W_{eff} = 4 × [(wl) / (2w + 2l)] or [4 × (pillar area / pillar perimeter)]

where w = Pillar width
l = Pillar length
h = Pillar height

with all dimensions in meters (1 meter (m) = 3.28 ft), and the pillar stress, due to overburden, is calculated using Equation 2.

The pillar strength is then compared to the pillar stress (using consistent units):

$$SF_{Stacey-Page} = \sigma_p / \sigma_v \qquad \text{(Eqn. 6)}$$

where $SF_{Stacey\text{-}Page}$ = Factor of safety in crushing by the Stacey-Page Method
σ_v = Pillar stress (in MPa) calculated using Equation 2

HARDY-AGAPITO METHOD

Hardy and Agapito (3) in the United States developed a pillar design method from observations in oil shale mines. "Oil shale" is neither "oil" nor "shale," but a hydrocarbon kerogen distributed in a fresh-water limestone. The pillar strength is estimated from:

$$\sigma_p = \sigma_c \left(\frac{V_s}{V_p} \right)^{0.118} \left[\frac{(w_p h_s)}{(w_s h_p)} \right]^{0.833} \qquad \text{(Eqn. 7)}$$

where σ_p = Pillar strength
σ_c = Compressive strength of a sample measured in the laboratory
V_s = Volume of sample tested
V_p = Volume of pillar
w_p = Width of pillar
h_s = Height of sample
w_s = Width of sample
h_p = Height of pillar

The pillar strength is then compared to the pillar stress as in

$$SF_{Hardy-Agapito} = \sigma_p / \sigma_v \qquad \text{(Eqn. 8)}$$

where $SF_{Hardy-Agapito}$ = Factor of safety in crushing by the Hardy-Agapito Method
σ_v = Pillar stress calculated using Equation 2

In most cases using NX- or NQ-sized diamond-drill core, the laboratory-tested sample dimensions are cylindrical with a nominal diameter of 2.0 inches and height of 4.0 inches.

OBERT-DUVALL METHOD

Obert and Duvall (4) proposed a pillar design method for rocks in general, but based mostly on strong rocks such as found in zinc and lead mines. The formulation is

$$\sigma_p = \sigma_c \, [0.778 + 0.222(w / h)] \qquad \text{(Eqn. 9)}$$

where σ_p = Pillar strength
σ_c = Unconfined compressive strength of a laboratory specimen
w = Pillar width and length
h = Pillar height

The pillar strength is then compared to the pillar stress as in

$$SF_{Obert\text{-}Duvall} = \sigma_p / \sigma_v \qquad \text{(Eqn. 10)}$$

where $SF_{Obert\text{-}Duvall}$ = Factor of safety in crushing by the Obert-Duvall Method
σ_v = Pillar stress calculated using Equation 2

Obert and Duvall (4, p. 458) recommend a safety factor of at least four for room-and-pillar mining with a checkerboard pattern using this method. The Obert and Duvall Method is thus either very conservative or unrealistic, depending on the point of view. It should be noted that the original application of this formula to pillar design was by Bunting (5) for use in the anthracite mines in northeastern Pennsylvania. Bunting reported that Johnson (6) derived the formula given in Equation 9 from laboratory strength data obtained by Bauschinger (7) for sandstone samples of different shapes. Close scrutiny of Bunting's paper shows that there was no attempt to account for reduction in strength of a pillar due to the effects of scale or discontinuities.

BIENIAWSKI/PENN STATE METHOD

Bieniawski (8) and Bieniawski, Alber and Kalamaras (9) proposed a pillar design method (also the CSIR or Penn State Method) for coal, based on research in South Africa and the United States, which is useful for weaker non-coal rocks. However, Agapito Associates, Inc. (AAI) experience is that the method is overly conservative in many hard-rock environments. The formulation is

$$\sigma_p = \sigma_1 \, [\, 0.64 + 0.36 \, (\, w / h \,) \,] \qquad \text{(Eqn. 11)}$$

where σ_p = Pillar strength
σ_1 = Unconfined compressive strength of a large cube of rock in the laboratory
or σ_1 = $C_o \; (d^{0.5}) / (\, 36^{0.5})$

where C_o = Laboratory-determined rock core unconfined compressive strength
d = Diameter of rock core
w = Pillar width and length
h = Pillar height

The pillar strength is then compared to the pillar stress as in

$$SF_{Bieniawski} = \sigma_p / \sigma_v \qquad \text{(Eqn. 12)}$$

where $SF_{Bieniawski}$ = Factor of safety in crushing for the Bieniawski Method

ABEL-WILSON-ASHWIN METHOD

Abel (10) developed the "Soft Rock Pillar Method" based upon the work of Wilson and Ashwin (11) that uses the concept of a confined core of a pillar which is the principal load-carrying element in the system; the confining restraint around the pillar core being provided by the broken rock resulting from the pillar excavation. The formulation for a wide rectangular pillar is (10)

$$L = \left(144 / 2000\right)\hat{\sigma}_v \left[pl - \left(p + l\right)\hat{y} + \left(4/3\right)\hat{y}^2\right] \qquad \text{(Eqn. 13)}$$

where L = load-carrying capacity in tons (2,000 lbs/ton)
$\hat{\sigma}_v$ = peak stable stress in pillar in psi = σ_h (tan B) + σ_o

where σ_o = pillar edge strength (psi)
σ_h = horizontal stress (psi)
tan B = $(\sin \varphi + 1\,) / (\, \sin \varphi - 1)$
[from Obert and Duvall (4, p. 288) as referenced by Wilson and Ashwin (10); note Abel (10) used B in this context while Obert and Duvall (4) and Wilson and Ashwin (11) used β]

where φ = angle of internal friction of pillar rock
$\hat{y}$ = yield zone thickness (ft)
= $\{h / [\, (\tan B)^{0.5} (\tan B) - 1)]\} \log_e (\, \hat{\sigma}_v / y\,)$

where h = pillar height (ft)
y = distance from pillar edge to confined core at point of interest (ft)
p = pillar width (ft)
l = pillar length (ft)

The load-carrying capacity is then compared to the load bearing on the pillar using the tributary area theory.

CRITIQUE

AAI has seen all of these methods in use (or misuse) in stone mine design and planning. Of these methods, we have found that the Obert-Duvall Method over-estimates the pillar strength considerably, yielding doubt as to its validity without using arbitrary discounting factors. The Bieniawski Method, developed for coal, must be used with caution in weaker sedimentary rocks, and has been found inappropriate by AAI for limestone and dolomite. The Abel-Wilson-Ashwin Method requires the incorporation of rock mechanics parameters that are difficult to measure or estimate such as the "pillar edge strength," the passive pressure coefficient, and the thickness of the material surrounding the confined core in hard rocks. The results of the Abel-Wilson-Ashwin Method are extremely sensitive to the values of these parameters, and users must be careful to understand the consequences of the outcomes.

For these reasons, I now use only the Hedley-Grant, Stacey-Page, and Hardy-Agapito methods (until something better comes along).

FIELD OBSERVATIONS

Recently, while examining existing shallow room-and-pillar stone mines, a few pillars were noted that had been excavated undersized. In one mine, the plan in place and performing successfully uses 45-ft by 45-ft square pillars with 45-ft wide rooms and 45-ft wide by 40-ft high crosscuts. The rock strength is approximately 20,000 psi, and the roof was at a depth of approximately 200 ft. The noted pillars were isolated, irregular, and approximately 15-ft square and 40-ft high, with the room and crosscut widths widened to 75 ft (full width, not half-width) to keep the same center-to-center distances. While there are no known in-situ rock stress measurements in the vicinity of the observed mine, a literature search revealed twenty in-situ stress measurements of various kinds within a 150 mile radius. When averaged, the horizontal stresses were approximately four times the vertical stress. However, no clear signs of horizontal stress effects were noted. Figure 1 shows the encountered situation in the mine. The undersized pillars exhibited slabbing, opening of through-going fractures, and hour-glassing, and were clearly distressed. Figure 2 shows a photograph of one such pillar.

Another stone mine of similar dimensions has two pillars undersized for unknown reasons that also exhibit distress. The mine is no longer operating and is at least 40 years old. The area is known to have high horizontal stresses. Some normal pillars have slabbed sides and corners. Interestingly, no clear horizontal stress effects were evident in these pillars, but the immediate roof exhibited shearing and cutter development.

Figure 3 is a plot of the Hedley-Grant, Stacey-Page, and Hardy-Agapito methods where the actual encountered pillar dimensions and conditions in the first mine are used to calculate safety factors. All three methods indicate that the 15-ft square pillar in the large rooms and crosscuts are near a safety factor of one. The original 45-ft square pillars and 45-ft wide rooms have very high safety factors and are clearly not strictly rock-mechanics stability-driven. Rather, I suspect, the pattern has been adopted from another mine and found successful, if not conservative. Rooms 45 ft wide are typical to accommodate large front-end loaders and haul trucks without overly-wide roof spans. Pillars planned at 30 or 35 ft in width will often be excavated even narrower by mine crews unless continual survey control and careful blasting techniques are adopted. In my opinion, in stone mines, pillars with width-to-height ratios significantly less than unity are to be avoided.

These three methods yield a safety factor of near one for this narrow pillar situation, which is indicative of distress. Had the calculated safety factors for the encountered narrow pillars been significantly higher than unity or significantly lower than unity, as indicated by the three methods, the result would be unrepresentative of encountered field in-mine conditions.

The meaning of a Factor of Safety of 1.0 is that the load brought to bear on the pillar is well-understood and properly characterized and that the strength of the mine rock has been correctly measured and projected to the pillar rock mass scale, conditions, and geometry, resulting in equilibrium. Such a circumstance of an understanding of *both* the load and the strength (ability to resist load) can be problematic. For instance, natural materials such as rock, in our experience, will vary in laboratory-measured strength by at least 25% from a mean strength. Practitioners will use something greater than unity for a factor of safety, based upon their understanding or their ignorance of the environment. I use different safety factors depending on the degree of acceptable risk.

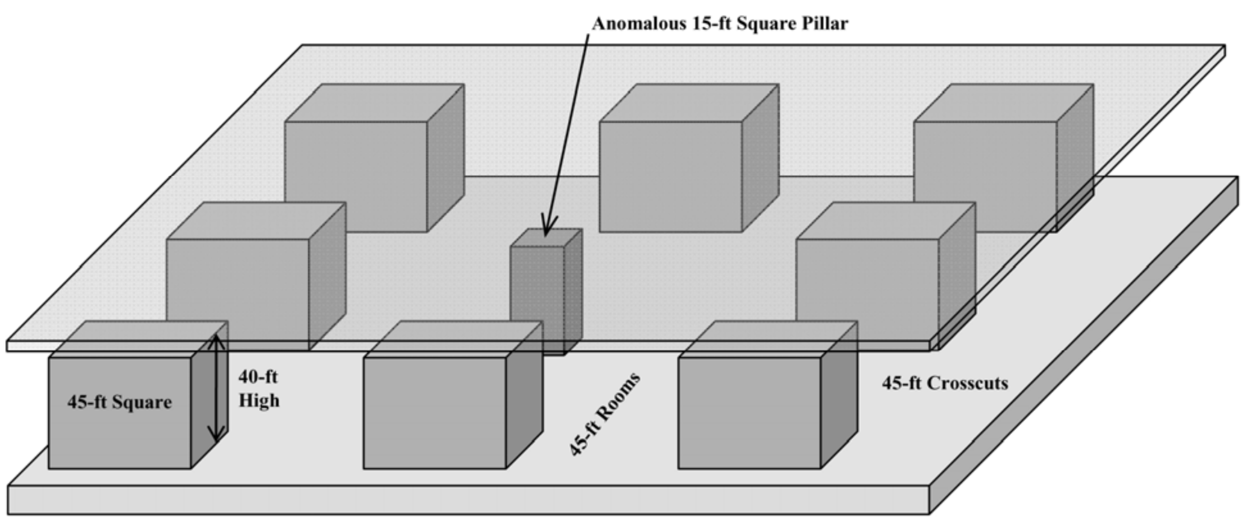

Figure 1. Idealized Isometric Drawing of Encountered Undersized Mine Pillar (not to scale)

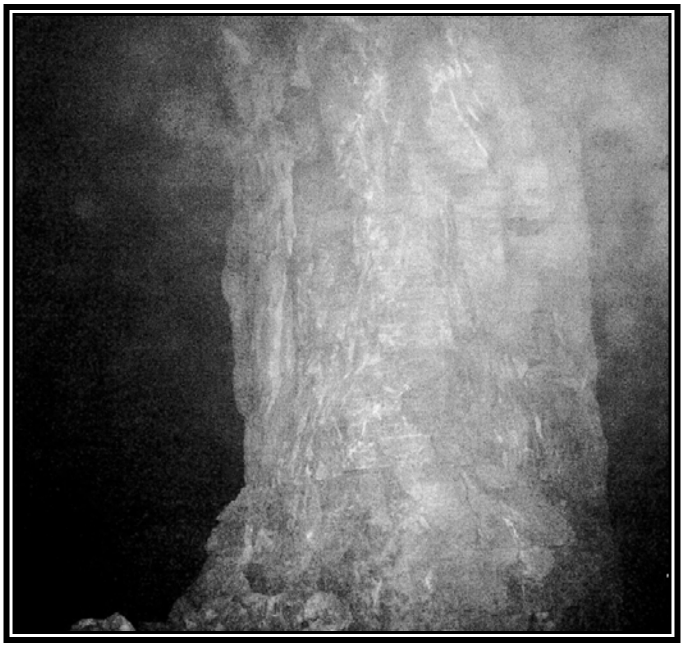

Figure 2. Photograph of Distressed Undersized Mine Pillar Encountered

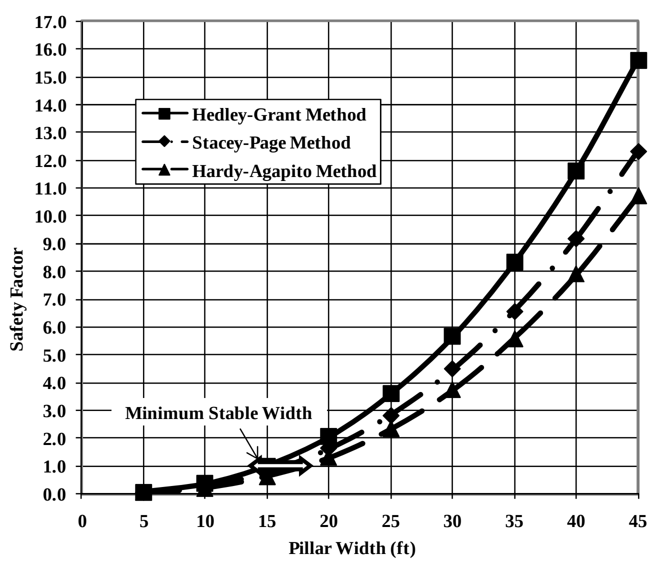

Figure 3. Calculated Safety Factors using Hedley-Grant, Stacey-Page, and Hardy-Agapito Methods

Without engaging in a lengthy and interesting discourse on rock mass loads, strength variability, and characterization, the pillar design practitioner should develop as good an understanding of the mine design environment as *practicable* (not *possible*—we will never have that degree of knowledge implied by *possible*, at least with foreseeable technological developments).

Iannacchionne (12) presents a summary of stone mine pillar performance based upon USBM and NIOSH field visits to United States mines. Iannacchionne also uses the Hedley-Grant, Stacey-Page, and Hardy-Agapito methods as examples of design practices, but points out the lack of data at higher pillar width-to-height ratios and at lower pillar width-to-height ratios for extending the use of these formulae. This present field study can supply a lower pillar width-to-height ratio data point of 15/40 or 0.375 for consideration in comparisons. The 0.375 width-to-height ratio is lower than any reported by Iannacchionne (12). For comparison with information presented by Iannacchionne (12), Figure 4 shows the calculated pillar strengths on this low 0.375 ratio (using a laboratory unconfined compressive strength of 20,000 psi), while Figure 5 shows the average pillar stress with 200-ft overburden for these same width-to-height ratios.

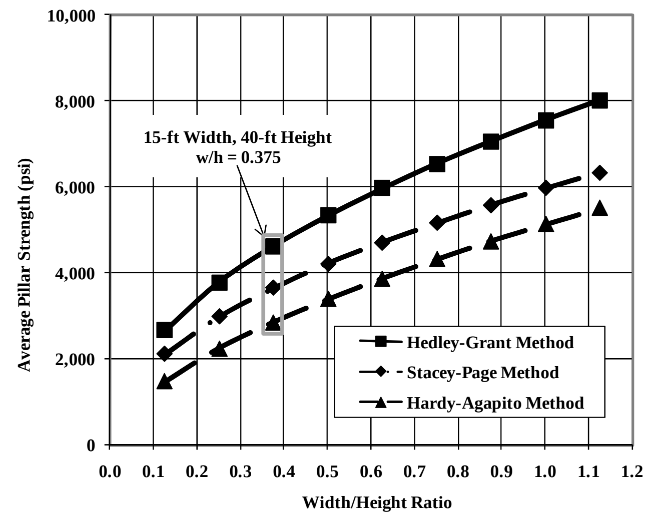

Figure 4. Calculated Pillar Strengths (divide psi by 145 to obtain MPa)

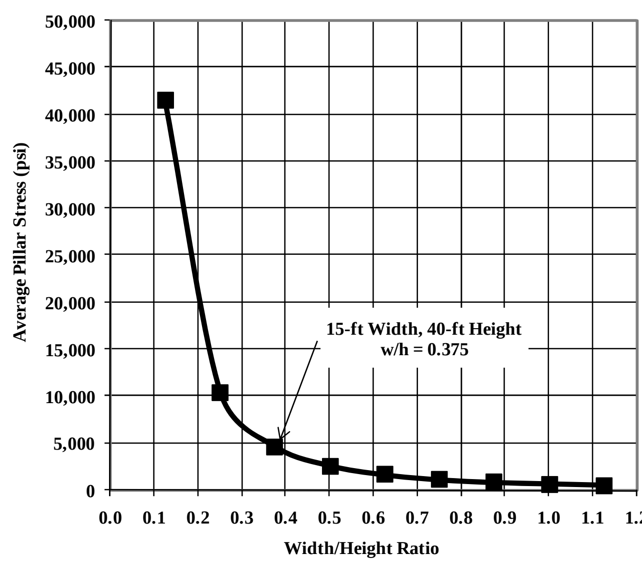

Figure 5. Calculated Pillar Stresses (divide psi by 145 to obtain MPa)

CONCLUSIONS

In conclusion, stone mine design methodology is workable if experienced practitioners utilize empirical formulations such as the Hedley-Grant, Stacey-Page, and Hardy-Agapito methods with a careful understanding of the origins and limitations of the methods. The agreement of the three methods for a narrow pillar that clearly exhibits distress characteristics and behavior is encouraging, and the practitioner should seek out case histories to further refine the methods.

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